Reduction to 1-body problem

Begin with the equations of motion for the 2-body problem

Simplify

Define

Then

Define the center of mass,

The path of the Center of Mass is a straight line and can be determined from initial conditions.

If is known, then and can be solved for.

Define the total mass, *M*, and , the reduced mass

Then,

If we treat as being the position of a body instead of a difference of positions, then we can interpret the above equation in the following way: A body with mass is gravitationally attracted to a body with mass whose position is fixed at the origin.

The value of doesn’t impact the motion of but it does ensure that the conserved quantities: angular momentum, and total energy are equal to those in the 2- body problem. In this way the 2-body problem is reduced to a 1-body problem.

Equation of Motion

To solve for the motion of we will be changing coordinates to a polar coordinate system. Since is conserved the motion of is planar and polar coordinates are sufficient to describe its motion.

The equation of motion in polar coordinates is derived as follows

and are perpendicular so

Where is the angular momentum and is constant.

Since total energy is conserved,

Note that is the centripetal force.

Solving

In the equation above we have a differential equation in time. Our goal is as a function of . To achieve this, we perform a change of variables

This equation can be transformed into a linear one by the following transformations.

Define,

Then,

Define,

Then,

Which is the form of an ellipse in polar coordinates that has one focus at the origin, has an eccentricity of , whose semi-major axis is , and is at an angle to the positive x-axis. Note that is at the angle of the apoapsis. It is analogous to the standard orbital parameter, the argument of periapsis. They both define the orientation of the ellipse in the orbital plane. is then analogous to the true anomaly.

Given the initial position and velocity it is possible to calculate and and therefore . See the file “solve for orbital parameters.docx” for details.